COMBINATORIAL RELATIONS IN THE EEG SPECTRUM

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The mean geometric frequency f'_1 of each EEG wave divides its frequency band $B = f_2 - f_1$ into subbands b and s. For the β wave the values of s, b, and B are elements of a geometric progression with denominator equal to the invariant F of the golden section. It is postulated that all EEG waves are characterized by an S_G system of recurrent equations, obtained by combinatorial generalization of a Fibonacci generating function. Theoretical invariants of the S_G system coincide with experimental b/s ratios with a standard error of 1%. The S_G system predicts the existence of ρ and σ EEG waves (55-118 and 118-225 Hz), which have not yet been found experimentally.

KEY WORDS: brain algorithms; biocybernetics; systems of biological equations.

In certain states of the brain, certain waves of the EEG are dominant; this probably reflects the optimization of brain structures when they perform a particular activity.

The object of this paper is to represent mathematically the characteristics of the EEG connected with optimization of brain activity.

EXPERIMENTAL METHOD

The processes lying at the basis of EEG formation are statistical in character. To determine the principles governing them, it is necessary to choose an informative parameter; to use a computer for automatic processing and averaging of the data, mathematical analysis of the results, and the putting forward of a theoretical hypothesis to account for all the results; to test the degree of congruence between theory and experiment.

EXPERIMENTAL RESULTS

Spectrograms of the spectral power of the EEG as functions of frequency, averaged in intervals of a few seconds (obtained by means of the computer), were found to be informative. They contain definite extrema, the number, magnitude, and frequency distribution of which depend on the functional state of the brain [1-3]. By analogy between electrical waves in the brain and an electronic generator of RC type and by quantitative analysis of these spectrograms it has been found [4] that extrema can appear only at mean geometric frequencies of bands and subbands of EEG waves. Agreement between the theoretical equation of the frequency distribution of the extrema and the experimental data (with an error of 4%) suggested the usefulness of a search for the principles connected with extrema of the spectral power.

The most interesting and important principles are those of the β wave, dominant during mental work, the most important function of the cerebral cortex. The boundary frequencies of the β wave in healthy adult persons [5, p. 53] are: $f_1 = 14$ Hz, $f_2 = 35$ Hz, hence $B = 21$ Hz; $f'_1 = (f_1 \cdot f_2)^{1/2} = \sqrt{14 \cdot 35} = 22.13$ Hz, $b = 12.87$ Hz, $s = 8.13$ Hz, and the ratio $(21/12.87) \simeq (12.87/8.13)$ differs only very little from the Fibonacci invariant $F = 1.618\ldots$, i.e., it obeys the law of the golden section:

$$\frac{B}{b} = \frac{b+s}{b} = \frac{b}{s} = F. \quad (1)$$

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TABLE 1. Generating Functions and Invariants

Set S_G of generating function	Set $E_C^S G$ of characteristic functions	Invariants I_λ (roots $E_C^S G$)
$a_3 = a_2 + a_1$	$\theta^2 - \theta - 1 = 0$	$I_\beta = F \simeq 1{,}618...$
$a_4 = a_3 + a_1$	$\theta^3 - \theta^2 - 1 = 0$	$I_\rho = (S\Phi)^{1/2} \simeq 1{,}465...$
$a_5 = a_4 + a_1$	$\theta^4 - \theta^3 - 1 = 0$	$I_\sigma = \Phi^{2/3} \simeq 1{,}380...$
$a_4 = a_2 + a_1$	$\theta^3 - \theta - 1 = 0$	$I_\theta = S \simeq 1{,}324...$
$a_5 = a_3 + a_1$	$\theta^4 - \theta^2 - 1 = 0$	$I_{\alpha,\gamma} = \Phi^{1/2} \simeq 1{,}272...$
$a_5 = a_2 + a_1$	$\theta^4 - \theta - 1 = 0$	$I_\Delta = (\Phi/S) \simeq 1{,}221...$

TABLE 2. Comparison of Experimental Data and Theoretical Invariants

Experimental data and theoretical invariants	EEG waves			
	θ	α	β	γ
$f_1 - f_2$ (in Hz)[5]	4—7	8—13	14—35	35—55
$f_1' = (f_1 \cdot f_2)^{1/2}$ (in Hz)	5,29	10,2	22,13	43,87
B (in Hz)	3	5	21	20
b (in Hz)	1,71	2,8	12,87	11,13
s (in Hz)	1,29	2,2	8,13	8,87
I_λ theoretical invariants	1,324	1,272	1,618	1,272
$R_1^{bc} = b/s$ experimental data	1,325	1,273	1,583	1,255
% difference between I_λ and R_1^{bs}	0,075	0,08	—2,21	—1,34
$I_\lambda^o = 1 + 1/I_\lambda$ theoretical invariants	1,755	1,786	1,618	1,786
$R_1^{Bb} = B/b$ experimental data	1,754	1,786	1,632	1,797
% difference between I_λ^o and R_1^{Bb}	—0,06	0,00	0,87	0,8

The β wave is thus the optimal characteristic ratio $R_1^{bS} = b/s$, equal to the invariant F.

Let us assume that all EEG waves accompanying intellecutal activity are described by an integral system S_G of equations (with invariant I_λ), each of which is obtained by means of a single combinatorial algorithm, and one of which is a Fibonacci generating function characterizing the golden section. The Fibonacci numbers F_n are a geometric progression with the denominator:

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \Phi \tag{2}$$

and the generating recurrent function:

$$F_n + F_{n+1} = F_{n+2} \tag{3}$$

with indices of neighboring numbers of the natural series. From Eq. (3) we find the generalization of the generating function of the golden section in the form of the set S_G of recurrent equations, formed in accordance with the rule, "The sum of consecutive elements a_i and a_j of the arithmetical progression is equal to the next element a_k:

$$a_i + a_j = a_k, \tag{4}$$

where the indices, i, j, and k are not necessarily the neighboring numbers of the natural series, $i < j < k$." If a_n are irrational numbers, then

$$\frac{a_{n+1}}{a_n} = I_\lambda = \text{Const}'' . \tag{5}$$

Since the invariant S is found in many living organisms [6-8] and since the principle of duality is manifested everywhere in nature [9], the set S_G must contain an even number of elements. The system S_G contains the minimal even number of equations for the combinatorial data: i = 1:. j = 2, 3, 4: k = 3, 4, 5 (Table 1).

The degree of congruence of the theoretical and experimental data is shown in Table 2.

The invariants I_λ are elements of the integral system S_G obtained with the aid of a single algorithm and connected with one another

$$\ln I_\lambda^{(n+1)} = \ln I_\lambda^{(n)} + (0.040 - 0.042). \tag{6}$$

It is clear from Table 2 that for θ, α, β, γ waves the theoretical invariants I_λ, I_λ^0 and the values of the experimentally obtained ratios b/s and B/b coincide with the standard error $\simeq 1\%$. This agreement lay at the basis of the writers' hypothesis of the existence of ρ and σ waves, not yet discovered experimentally. The S_G system contains invariants I_ρ and I_σ, and it thus predicts the existence of ρ and σ waves (55-118 and 118-225 Hz).

LITERATURE CITED

1. A. M. Klochkov, L. I. Sergeeva, and P. A. Elkin, "Comparative assessment of stochastic characteristics of electroencephalograms," Zh. Vyssh. Nerv. Deyat., No. 2, 377 (1967).
2. A. M. Klochkov, "Comparative assessment of the informativeness of the spectral characteristics of the encephalogram and its derivative," Zh. Vyssh. Nerv. Deyat., No. 6, 1011 (1969).
3. A. M. Klochkov, P. A. Elkin, and V. D. Zheleznyakov, "The use of spectral characteristics of the EEG and its derivative in practical aviation physiology," Zh, Vyssh. Nerv. Deyat., No. 2, 560 (1971).
4. A. A. Sokolov, P. P. Kondrat'ev, and Ya. A. Sokolov, "Dual principles in the EEG spectrum," Dokl. Akad. Nauk SSSR, 205, 1473 (1972).
5. P. I. Gulyaev, Electrical Processes in the Human Cerebral Cortex [in Russian], Leningrad (1960).
6. A. Zeising, Aestetische Forschungen, Frankfurt (1855).
7. T. A. Cook, The Curves of Life, London (1914).
8. D. Thompson, On Growth and Form, Cambridge (1942).
9. P. Curie, Selected Works [Russian translation], Moscow, Izd. Akad. Nauk SSSR (1966).